

The primary reason the "quadratic formula" is used is to help solve quadratic equations that are not easily factorable. It can be used anytime to find roots.

Here are some examples.

Equations that are easily factorable	Equations that aren't easily factorable
$x^2 - 5x - 6 = 0$	$x^2 - 3x - 11 = 0$
$6x^2 + 4x - 2 = 0$	$6x^2 + 7x - 7 = 0$

Given a quadratic equation in general form, we CTS to derive the quadratic formula

<ul style="list-style-type: none"> • $ax^2 + bx + c = 0$ • $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ • $\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + \frac{c}{a} - \left(\frac{b}{2a}\right)^2 = 0$ • $\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2 = 0$ • $\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$ • $x + \frac{b}{2a} = \pm \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$ • $x = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$ • $x = -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$ • $x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$ • $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ • $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 	<ul style="list-style-type: none"> • Divide equation by 'a' to isolate 'x²' • Complete the square • Isolate 'x', in order to find the roots • Don't forget the square root of any number will give two possible answers • Any quadratic equation in general form, $ax^2 + bx + c = 0$, where $a, b, & c$ are coefficients, $a \neq 0$, and $b^2 - 4ac \geq 0$, we can always use the quadratic formula to find its roots
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Example 1: Use the QF to determine the roots for the following equations

a) $x^2 - 3x - 11 = 0$	b) $6x^2 + 7x - 7 = 0$
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There are a few conditions when using the QF

1. One side of the equation must equal zero, so all terms must be on one side

$$(x+2)(2x-3) = 5x+1 \longrightarrow (x+2)(2x-3) - 5x - 1 = 0$$

2. The equation must be simplified and in general form ($ax^2 + bx + c = 0$)

$$(x+2)(2x-3) - 5x - 1 = 0$$

$$(2x^2 + x - 6) - 5x - 1 = 0$$

$$2x^2 - 4x - 7 = 0$$

3. The number under the square root must be positive, i.e., $b^2 - 4ac \geq 0$, since the square root of a negative number can't occur, therefore no possible roots

$$(-4)^2 - 4(2)(-7) \geq 0$$

Example 2: Use the QF to determine the roots for the following equations

c) $x^2 - 3x - 11 = 0$	d) $6x^2 + 7x - 7 = 0$
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Example 3: The revenue generated by Small Business Corporation is given by the equation $R = -20(p - 20)^2 + 3000$, where ' p ' is the price of the product. Determine the range in price that will yield revenue in excess of \$3000.

Example 4: The bus of the Vancouver Giants travelled 720 km from Vancouver (V) to Portland (P). On the trip home, the average speed increased 10 km/h. If the round trip took 17 hours, what was the average speed of the bus? ($d = v \times t$)

Trip	Distance (km)	Speed (km/h)	Time (h)
V to P			
P to V			

When referring to the "nature of the roots", we are talking strictly about the number of roots that a quadratic equation has, i.e., all quadratic equations will have one of the following number of roots:

1. 2 distinct real roots (2 different x-intercepts)
2. 2 equal real roots, aka, "double root" (1 x-intercept)
3. No real roots (no x-intercepts)

To determine the "nature of the roots" for any quadratic equation, use only the "discriminant" ($b^2 - 4ac$) from the QF, i.e., $x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$. Ignore the radical.

The following will determine the number roots:

1. 2 distinct real roots if: $b^2 - 4ac \geq 0$
2. 1 distinct real root if: $b^2 - 4ac = 0$
3. No real roots if: $b^2 - 4ac < 0$

Example 5: Determine the nature of the roots

a) $2x^2 + 3x - 10 = 0$	b) $9x^2 - 12x = -4$	c) $3x^2 = 7x - 5$
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Example 6: The height, $h(t)$, of an object thrown upward from the top of an 80 m cliff at 20 m/s is related to its time, t , by the function $h(t) = -5t^2 + 20t + 80$. Will the object reach a height of 100 meters? If so, at what time(s)?

Homework: